Lecture 1

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Samples from X

$X_1, X_2, \ldots, X_n$ → To find the distribution of X, i.e., the values X can take with its probability

↓

discrete finite data points

↓

that means the number of values X_i can take are finite

↓

It we need to gain some information

Now suppose we have sample way more than or enough amount of to get a population for X.

↳ To get a good estimate of $\mathbb{P}(X_i = val)$ $val \in Dis(X)$

↓

Now we can do a density plot and get an idea of $\mathbb{P}(X = val)$.

Now if X is continuous → we can discretize it into states.

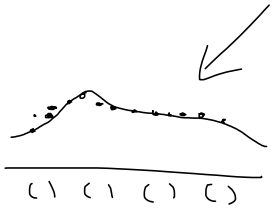
Example – $X \in [-1, 1]$ —after discretizing→ states: $[-1, -\frac{1}{3}), [-\frac{1}{3}, \frac{1}{2}], (-\frac{1}{2}, 0), [0, \frac{1}{2}], (\frac{1}{2}, 1]$

From samples,

Now we can get estimate of $\mathbb{P}(X \in state_i)$ but not the Distribution of X

Basically we want density $\hat{f}(X)$ or cumulative $F(X)$.

↳ There are many methods like Smoothenging, fitting fuction,



After getting an idea on how the distribution looks we can do the statistical tests like normal or skewness or HSIC tests to get an mathematical formulation of the $f(X)$ ↓ Hilbert Schmidt

Def of Optimation Problem :- (OP)

Given a fuction $f : S \to \mathbb{R}$ we ned to find $\left(\arg\min_{x \in S} f(x)\right)$.

- f: objective fuction
- S: domain or solution set
- $\mathbb{R}$: co domain
- $f(S)$ is the range, $f(S) \subseteq \mathbb{R}$
- x: is the design vaicble

Components of OP

constraints on the disign vaicble x

If any one of componets of op is not defined properly we say op is ill-posed

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Categorize the OP s :-

Some exmpls of Methods

- Continuous
  - smooth → Gradient Descnt, Momentum, Gauss-Newton,
  - non-smooth → Proximal Gradient, Chambolle-Pock
  - constrained → KKT, Linear Pragrams,
- Discrete / Combinatorial
  - Binary → Markov Random Fields,
  - Multi
  - Graphical → α-expansion, Graph Cuts, min-cost flow
- Variational → Partial Derivative Equation → PDE, prime-dual methods, Total Vaiation, Stochastic Methods.
